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The dark side of curvature

Gabriela Barenboim

Departament de Física Teòrica,Universitat de València and IFIC
Carrer Dr. Moliner 50, E-46100 Burjassot (València), Spain
Gabriela.Barenboim@uv.es

Enrique Fernández Martínez

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
Föhringer Ring 6, 80805 München, Germany
enfmarti@mppmu.mpg.de

Olga Mena

Instituto de Física Corpuscular, IFIC, CSIC and Universidad de Valencia, Spain
omena@fnal.gov

Licia Verde

ICREA & Instituto de Ciencias del Cosmos, Universitat de Barcelona, Marti i Franques 1, 08028, Barcelona, Spain liciaverde@icc.ub.edu

Abstract: Geometrical tests such as the combination of the Hubble parameter $H(z)$ and the angular diameter distance $d_A(z)$ can, in principle, break the degeneracy between the dark energy equation of state parameter $w(z)$, and the spatial curvature Ω_k in a direct, model-independent way. In practice, constraints on these quantities achievable from realistic experiments, such as those to be provided by Baryon Acoustic Oscillation (BAO) galaxy surveys in combination with CMB data, can resolve the cosmic confusion between the dark energy equation of state parameter and curvature only statistically and within a parameterized model for $w(z)$. Combining measurements of both $H(z)$ and $d_A(z)$ up to sufficiently high redshifts $z \sim 2$ and employing a parameterization of the redshift evolution of the dark energy equation of state are the keys to resolve the $w(z) - \Omega_k$ degeneracy.

Keywords: .

Contents

1. Introduction

Current cosmological measurements point to a flat universe whose mass-energy includes 5% ordinary matter and 22% non-baryonic dark matter, but is dominated by a dark energy component, identified as the engine for accelerated expansion e.g., [1, 2, 3, 4, 5, 6, 7, 8]. The present-day accelerated expansion reveals new physics missing from our universe's picture, and it constitutes the fundamental key to understand the fate of the universe.

The most economical description of the cosmological measurements attributes the dark energy to a Cosmological Constant in Einstein's equations, representing an invariable vacuum energy density. The equation of state parameter of the dark energy component in the cosmological constant case is constant, $w = \rho/p = -1$. A dynamical option is to suppose that a cosmic scalar field ϕ, called quintessence, changing with time and varying across space, is slowly approaching its ground state. In the quintessence scenario the equation of state is given by $w = (\dot{\phi}^2/2 - V(\phi))/(\dot{\phi}^2/2 + V(\phi))$ and in general it is not constant through cosmic time [9, 10, 11, 12, 13, 14]. Another alternative is that the dark energy is an extra cosmic fluid with a complex (time dependent) equation of state parameter $w(z)$.

However, given the fact that we only know of dark energy from its gravitational effects, what we are trying to explain by the addition of exotic fluids could just simply be explained by corrections to Einstein gravity e.g., [15, 16, 17, 18]. Although this requires a modification of Einstein's equations of gravity on large scales, this is not unexpected for an effective 4-dimensional description of higher dimensional theories. Most of the modifications of gravity proposed so far are based either on models with extra spatial dimensions or on models with an action which is non linear in the curvature scalar (that is, higher derivative theories, scalar-tensor theories or generalized functions of the Ricci scalar).

Determining the nature of dark energy is among the major aims of future galaxy surveys. A mandatory first step is to extract as precisely as possible the dark energy equation of state and its time dependence. Current cosmological limits on the equation of state parameter are model-dependent. In particular, most of the reported limits rely on the assumption of an underlying spatially flat, $\Omega_k = 0$ universe.

In this paper we focus on the cosmic confusion between the equation of state parameter w and a non-negligible spatial curvature Ω_k, exploring both constant w and redshift dependent $w(z)$ cases. Namely, it is quite possible that the universe we live in could be of the ΛCDM-type with a small curvature component. In such a universe, if the curvature is assumed to be zero, one would reconstruct a $w \neq -1$. Thus, by combining data at different redshifts, the equation of state reconstructed under the incorrect assumption of zero curvature, could be a time dependent $w(z)$. The authors of [19] study the degeneracy between Ω_k and w, for constant w. By exploiting luminosity distance data at different redshifts, they identify a critical redshift z_{cr}(which turns out to be ~ 3) at which the the luminosity distance becomes insensitive to curvature and the error on w is minimal (see also Ref. [20]). They conclude that the degeneracy between Ω_k and w could be alleviated if one combines luminosity distance data at redshifts below and above z_{cr}, since the $\Omega_k - w$ degeneracy at $z < z_{cr}$ is opposite to that at $z > z_{cr}$. The authors of [21] extended the previous analysis, considering dynamical dark energy models $w(z)$, and other fundamental observables, such as the Alcock-Pazynski test [22]. More recently, it has been shown [23] that the $w(z)$ reconstructed assuming zero curvature from Hubble parameter $H(z)$ will have a divergence if the curvature is negative; conversely if the curvature is positive it is the $w(z)$ reconstructed from the angular diameter distance $d_A(z)$ that will have a divergence. The redshift position of the divergence depends on the size of Ω_k. Thus, in principle, the different behaviours for these two reconstructed $w(z)$'s could be used to infer both the sign and the size of the cosmic curvature and the dynamical character of the dark energy component.

Here we show that, when realistic errors on $H(z)$ and $d_A(z)$ expected from future BAO surveys are considered, this is not possible: the expected signal is smaller than the errors. It is still possible to separate the effects of curvature from those of dark energy, but it must be done statistically, within a parameterized model for $w(z)$. We also forecast, using the Fisher matrix formalism, the errors on $w(z)$ (parameterized by a popular 2-parameter model) and Ω_k using measurements from a variety of surveys with characteristics not too dissimilar from those of planned spectroscopic and photometric galaxy surveys, in combination with forecasted constraints from a CMB experiment with characteristics similar to those of the Planck mission. We quantify the benefits of increased volumes and, in the case of photometric surveys, reduced photo-z errors. We show that in all these cases, the $w(z)$-Ω_k degeneracy is greatly alleviated if the BAO surveys cover a redshift range up to z$\sim$ 2.

The structure of the paper is as follows. In Sec. 2 we present the constraints on $w(z)$ and Ω_k from current available data. We present the reconstructed $w(z)$ from $H(z)$ and $d_A(z)$ mock data and errors in Sec. 3. Section 4 is devoted to the future Ω_k and $w(z)$ constraints from a variety of surveys which will cover different volumes and different redshift ranges. These surveys could provide the ideal tool to pin down both the cosmic curvature and measure the dark energy simultaneously. We conclude in 5.

2. Current Cosmological Constraints on Ω_k and $w(z)$

In this section we explore the current constraints on both a constant and a two-parameter model for the dynamical dark energy equation of state $w(z)$, assuming a non zero spatial curvature (see also Ref. [24, 25, 26]). We work in the framework of a cosmological model described by nine free parameters [1],

$$\theta = \{w_b, w_{dm}, \theta_{CMB}, \tau, \Omega_k, n_s, w_0, w_a, A_s\} \ , \tag{2.1}$$

being $w_b = \Omega_b h^2$ and $w_{dm} = \Omega_{dm} h^2$ the physical baryon and dark matter densities respectively [2], θ_{CMB} [3] a parameter proportional to the ratio of the sound horizon to the angular diameter distance, τ the reionisation optical depth, Ω_k the spatial curvature, n_s the scalar spectral index and A_s the scalar amplitude. The parameterization of the dark energy equation of state we use here, in terms of the scale factor a, reads

$$w(a) = w_0 + w_a \ (1 - a) \, , \tag{2.2}$$

which has been extensively explored in the literature [27, 28, 29, 30]. In terms of the redshift, Eq.(2.2) reads

$$w(z) = w_0 + w_a \ \frac{z}{1+z} \ . \tag{2.3}$$

We chose this parameterization because, in the absence of observational indications that $w(z)$ is not constant, it has become the standard one to use by all authors to be able to compare constraints obtained by different analysis, using different data. Of course, should any indication of a redshift-dependent $w(z)$ arise, the type of parameterization assumed would need to be a realistic fit to the data for the results to be meaningful. For the numerical simulations presented in this section we will assume the priors $-2 < w_0 < 0$ and $-1 < w_a < 1$. In this work we use the publicly available package `cosmomc` [31]. The code has been modified [32] for the time dependent $w(z)$ case.

On the data side, we start with a conservative compendium of cosmological datasets. First, in what we call *run0*, we include WMAP 5-year data [3, 4] and a prior on the Hubble parameter of $H_0 = 74.2 \pm 3.6$ km/s/Mpc from Ref. [33]. We then add in *runI* the constraints coming from the latest compilation of supernovae (SN) from Ref. [34]. Finally, we use the data on the matter power spectrum LSS from the spectroscopic survey of Luminous Red Galaxies (LRGs) from the Sloan Digital Sky Survey (SDSS) survey [6] which we refer to as the LSS data (*runII*). In summary,

- *run0*=WMAP(5yr)+H_0
- *runI*=*run0*+SN
- *runII*= *runI*+LSS

[1] We use the full set of parameters with both w_0 and w_a when considering a varying $w(z)$, and a reduced set with $w = w_0$ and $w_a = 0$ when considering the constant w case.

[2] The current value of the Hubble parameter H_0 is defined as $100h$.

[3] The θ_{CMB} parameter can be replaced by the H_0 parameter. However, using θ_{CMB} is a superior choice due to its smaller correlation with the remaining parameters.

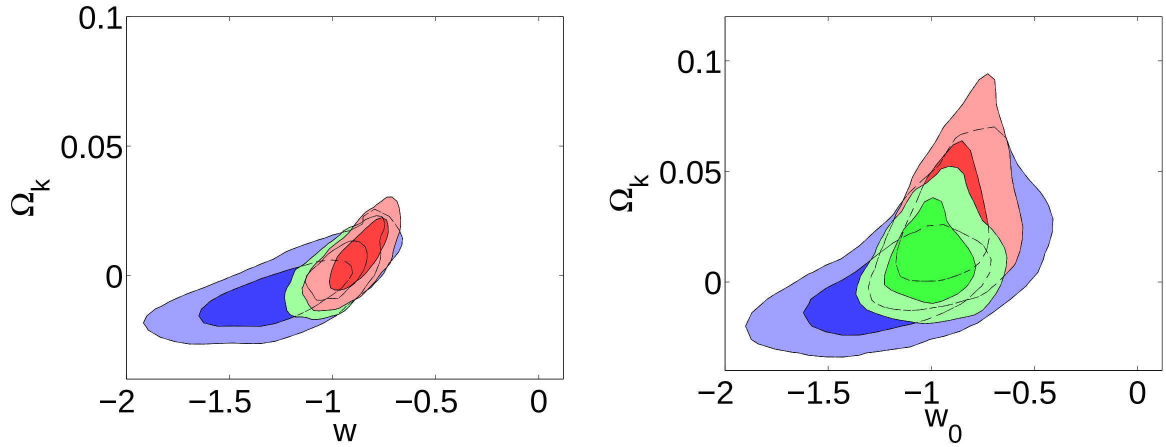

Figure 1: *Left panel: 1 and $2-\sigma$ constraints on the w-Ω_k plane for a constant w model. The blue, green and red contours show the $1-$ and $2-$ σ joint confidence regions (marginalised over all other parameters) for run0 and runI both combined with SDSS BAO data, and for runII results, respectively. Right panel: 1 and 2σ constraints on w_0-Ω_k plane for the full 9-parameters model where $w(z)$ is parameterized by w_0 and w_a as in Eq. 2.2.*

We combine both *run0* and *runI* results with SDSS BAO data [35] at $z = 0.35$. We first consider the case of a constant equation of state w. Figure 1 (left panel) shows the 1 and 2σ joint constraints in the w-Ω_k plane from current data (marginalised over all other 6 parameters). We show the results from the three runs described above. Notice that a degeneracy is present, being w and Ω_k positively correlated. The shape of the contours can be easily understood. In a universe with a dark energy component with a $w > -1$ the distance to the last scattering surface will be shorter, effect which can be compensated in an open universe with $\Omega_k > 0$. The opposite happens if $w < -1$. A similar analysis to the one shown in Fig. 1 (left panel) is presented in Ref. [4]. Here we use a prior on θ_{CMB} rather than on H_0 (as done in Ref. [4]). Notice however that the tendency of the degeneracy in the $w - \Omega_k$ plane is the same in both studies.

Next, we explore the case of a time dependent $w(z)$. The parameterization we use is given by Eq. (2.3). The right panel of Fig. 1 shows the constraints in the w_0-Ω_k plane. Notice that the constraints on both the spatial curvature Ω_k and w_0 are much weaker than those obtained when we assume a constant w, allowing for much larger positive values of Ω_k (which correspond to an open universe). We obtain as well less stringent constraints on w_0 than those obtained on w due to the addition of the extra w_a parameter. If one relaxes the assumption of constant equation of state, the 2σ marginalized error on Ω_k is ~ 0.03 for *runI* plus SSDS BAO data and 0.04 for *runII*. Comparing to the constant w case this corresponds to an increase of a factor ~ 1.8 and 2.3 respectively in the errors on Ω_k. Notice that, even after combining with BAO data, if Ω_k is positive (open universe), for a model with time dependent $w(z)$, the maximum allowed cosmic curvature contribution is double of that allowed for a model with a constant w. The contours for the constant w and non constant w cases are very similar in the $\Omega_k < 0$ region, since Ω_k can not be

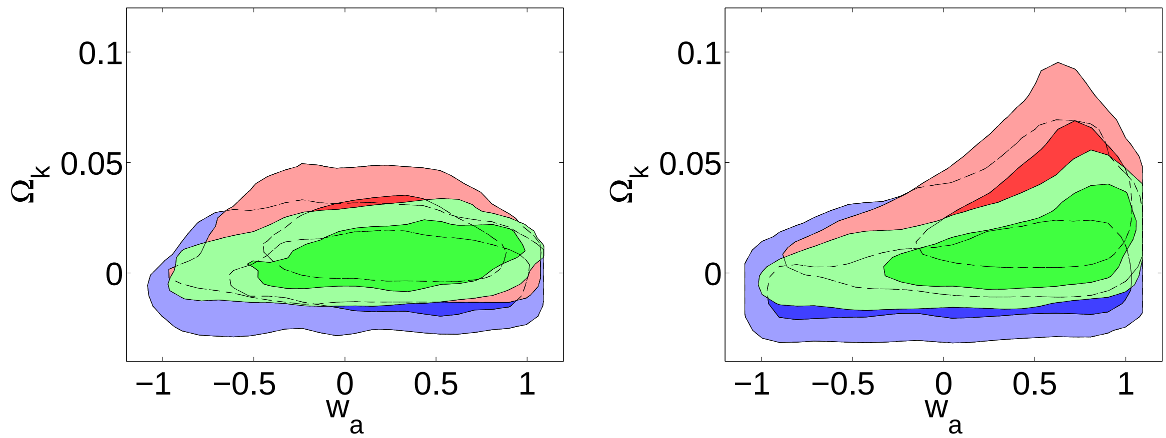

Figure 2: *1 and $2-\sigma$ constraints on the w_0-Ω_k plane for the full 9-parameters model. The blue, green and red contours depict run0, runI (both combined with SDSS BAO data) and runII results, respectively. Left panel: including dark energy perturbations Right panel: without including dark energy perturbations in the analysis.*

arbitrarily small (the total energy density parameter including the curvature contribution needs to be 1).

Figure 2 shows the 1 and 2σ constraints in the w_a-Ω_k plane for the full 9-parameters model. The right panel allows for perturbations in the dark energy while in the left panel perturbations are switched off. With current data we are unable to produce meaningful constraints on the w_a parameter. Notice that the contours are closed by the prior imposed ($-1 < w_a < 1$). It is also evident that the effect of the dark energy perturbations is important if the universe is open, i.e. $\Omega_k > 0$. Analogously to what happens in the flat, constant w case [4], the addition of the dark energy perturbations into the analysis changes the allowed parameter space, lessening the constraints in the w_a-Ω_k plane.

It is well known that crossing the phantom divide $w = -1$ can lead to divergences in the dark energy perturbation equations [36, 37], for a fixed dark energy sound speed c_s^2. Also, dark energy perturbations may not be well physically well defined for models where $w < -1$; for example modified gravity models or k-essence, can yield effectively $w < -1$. Some works in the literature switch off the perturbations when $w < -1$. [38] argues that for physical solutions and to avoid instabilities and ghosts, the sound speed should be zero– or small and negative- for $w < -1$. Here we follow the treatment presented in Ref. [32], which provides a method to cross $w = -1$ and avoids instabilities. In addition, in the absence of a fully motivated and specified dark energy model, we simply show both cases, with and without perturbations, to quantify their effect.

3. The $w - \Omega_k$ degeneracy and future BAO surveys

Acoustic oscillations in the photon-baryon plasma are imprinted in the matter distribution.

[4]The authors of Ref. [2] conclude that, when the dark energy perturbations are considered, the (tiny) extra dark energy clustering reduces the size of the quadrupole and therefore its power to extract w.

These Baryon Acoustic Oscillations (BAO) have been detected in the spatial distribution of galaxies by the SDSS [35] and the 2dF Galaxy Reshift Survey [39, 40]. The oscillation pattern is characterized by a standard ruler, s, whose length is the distance sound can travel between the Big Bang and recombination and at which the correlation function of dark matter (and that of galaxies, clusters) should show a peak. Detecting this scale s at different redshifts is the major goal of future galaxy surveys.

Therefore, the aim of a BAO survey is to measure the location of the baryonic peak in the correlation function along ($s_{\parallel} = \Delta z$) and across ($s_{\perp} = \Delta\theta$) the line of sight. In the radial direction, the BAO directly measure the instantaneous expansion rate $H(z)$ through $s = (c/H(z))s_{\parallel}$, where $H(z)$ is given by the Friedmann equation,

$$H^2(z) = H_0^2 \left(\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right) \tag{3.1}$$

where Ω_m, Ω_k and $\Omega_\Lambda = 1 - \Omega_m - \Omega_k$ are the energy density of the universe in the form of dark matter, spatial curvature and dark energy respectively.

At each redshift, the measured angular (transverse) size of oscillations, $s_{\perp}$, corresponds to the physical size of the sound horizon, $s(z) = d_A(z)s_{\perp}$, where the angular diameter distance d_A reads

$$d_A(z) = \frac{1}{H_0\,\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k}\int_0^z dz' \frac{H_0}{H(z')}\right) \ , \tag{3.2}$$

which is formally valid for all curvatures, and $H(z)$ is given by Eq.(3.1). The fact that, given sufficient redshift precision, a BAO survey can measure both the $H(z)$ component ($s = (c/H(z))s_{\parallel}$) and the transversal component offers a powerful consistency check: the recovered $H(z)$ must agree with the recovered $d_A(z)$, which is an integral of $1/H(z)$. This feature is the key to disentangle curvature from dark energy properties as we will illustrate next.

There are future large scale surveys such as BOSS[5], Euclid[6], JDEM[7] and LSST[8] planned, which will cover $\mathcal{O}(10000)$ square degrees of the sky and are expected to extract the angular extent of the BAO signature and, redshift precision allowing, also the BAO feature in the radial direction. Therefore, these future surveys are expected to provide measurements of $d_A(z)$ (and, in many cases of $H(z)$) in the $z \lesssim 3$ redshift interval. In the next subsection we discuss how in principle, reconstructing in a model-independent way a time dependent dark energy equation of state $w(z)$ from measurements of $H(z)$ and $d_A(z)$ independently, could help to identify the presence of cosmic curvature [23]. However, when realistic errors from future surveys are considered, this procedure fails. The expected errors in $d_A(z)$ and $H(z)$ can be estimated using, for instance, the Fisher matrix approach presented in Ref. [41]. Similar results could be obtained using the errors forecasted in Ref. [42].

[5]http://www.sdss3.org/cosmology.php

[6]http://sci.esa.int/science-e/www/area/index.cfm?fareaid=102

[7]http://jdem.gsfc.nasa.gov/

[8]http://www.lsst.org/

3.1 Reconstructing $w(z)$ via measurements of $H(z)$ and $d_A(z)$

We start by following Ref. [23]. Let us assume that the universe is such that there is a small curvature component and a cosmological constant. In such a universe the Hubble parameter is given by the expression:

$$H^2(z) = H_0^2 \left(\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda \right) \ . \tag{3.3}$$

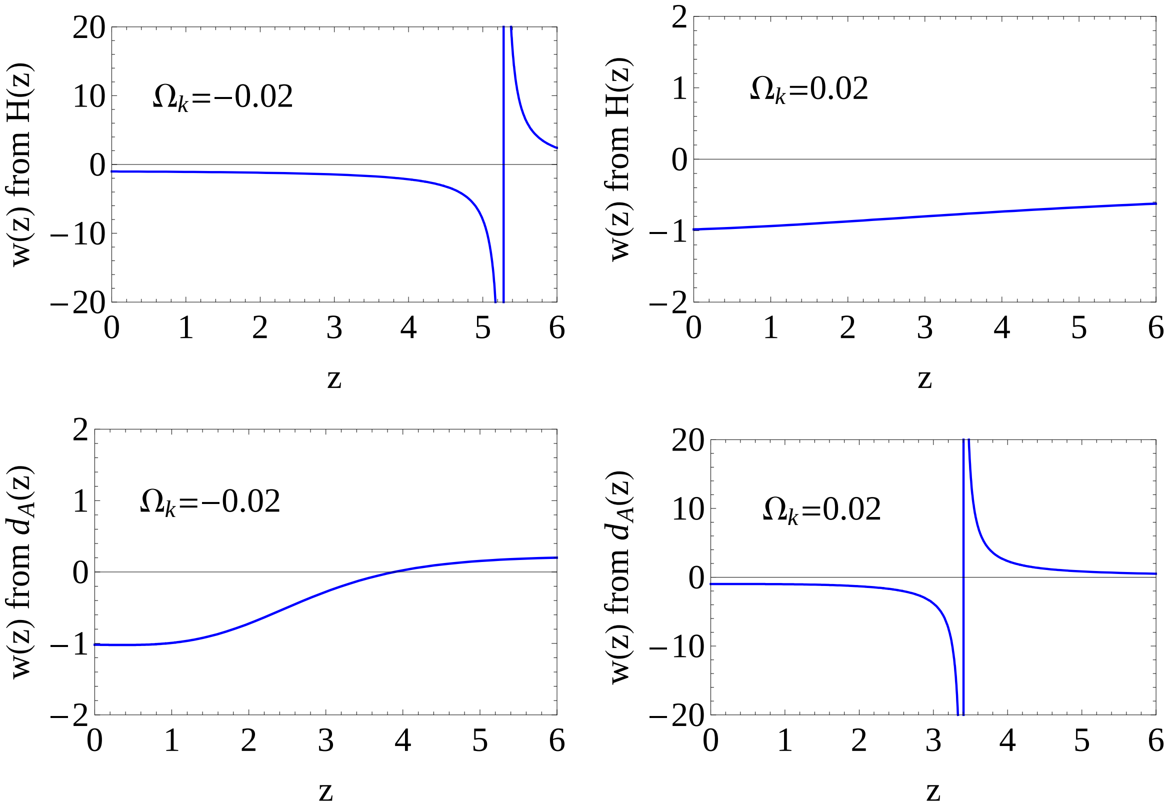

Figure 3: *In an ideal case with perfect measurements for the radial and angular BAO location as function of redshift, the inferred $w(z)$ from $H(z)$ and $d_A(z)$ under the assumption of flatness do not coincide in the presence of curvature. The top panels show the $w(z)$ inferred from $H(z)$: on the left is the case where a LCDM negatively curved universe is erroneously assumed to be flat and on the right is the negatively curved case. The bottom panels show the equivalent situation for when $w(z)$ is inferred from $d_A(z)$. In all panels $|\Omega_k| = 0.02$.*

If $H(z)$ could be perfectly measured, and used to reconstruct $w(z)$ but assuming zero curvature, the inferred $w(z)$ would be the following function of the Hubble parameter and its first derivative [23]

$$w_H(z) = -\frac{1}{3} \frac{2(1+z)HH' - 3H^2}{H_0^2 \Omega_m (1+z)^3 - H^2} \ , \tag{3.4}$$

where $' = \mathrm{d}/\mathrm{d}z$ and the Hubble parameter H is given by Eq.(3.3).

If one focused instead on the angular diameter distance and its first and second derivatives, the $w(z)$ reconstructed would be given by

$$w_{d_A}(z) = \frac{-3d'_A - 2(1+z)d''_A}{3\left(1 - (1+z)^3\Omega_m d'^2_A\right) d_A} \tag{3.5}$$

Figure 3 depicts the $w(z)$ that would be inferred after substituting in Eq.(3.4) and in Eq.(3.5) the value of the Hubble parameter, the value of the angular diameter distance and their derivatives versus redshift for a non flat universe with a cosmological constant.

For the example illustrated in Figs. 3, we assume $|\Omega_k| = 0.02$. Notice that, for negative curvature, while the $w(z)$ inferred from $H(z)$ and $H'(z)$ tends to values $w < -1$ as the redshift increases, the $w(z)$ reconstructed from the angular diameter distance and its derivatives tends to values $w > -1$. For positive curvature the behaviour of the $w(z)$ reconstructed from the angular diameter distance and the $w(z)$ inferred from $H(z)$ is the opposite.

An incorrect assumption about the Universe geometry would therefore show up as an inconsistency between the radial and transverse BAO. After combining measurements of $H(z)$ and $d_A(z)$ one would naively expect to break the $w(z)$-curvature degeneracy and to reconstruct an equation of state which resembles the underlying *true* cosmology, i.e. $w = -1$.

Perhaps the most attractive feature to attempt such a reconstruction lies in the fact that the inferred $w_H(z)$ and $w_{d_A}(z)$ exhibit a resonant-like behaviour, i.e. only one of the reconstructed w's will diverge depending on the sign of the curvature and the position of the pole will signal the size of this curvature. A negative curvature will be indicated by a resonant behaviour in the inferred $w_H(z)$ with a pole at a redshift of

$$z = \sqrt{-\Omega_\Lambda/\Omega_k} - 1\ , \tag{3.6}$$

while a positive curvature will make $w_{d_A} \to \infty$ at z satisfying the condition

$$H_0^2(1+z)^3\Omega_m \cosh^2\left(\sqrt{\Omega_k}\int \frac{H_0}{H(z')}dz'\right) = H^2(z)\,. \tag{3.7}$$

Therefore, one can be lead to believe that such a striking feature cannot be missed and envision that the degeneracy between $w(z)$ and Ω_k can easily be lifted. Unfortunately, for realistically achievable constraints on $H(z)$ and $d_A(z)$ this will not be the case as we will show next.

In order to mimic future $H(z)$ and $d_A(z)$ data, we have assumed a very optimistic survey, similar to an LSST-type survey but with no photo-z errors, with a volume of 30000 squared degrees and a redshift range from $z = 0.3$ up to $z = 3.6$, in bins of $\Delta z = 0.1$ width. The mean galaxy density is chosen to be $n = 3\times10^{-3}$. After generating mock data for $H(z)$, $d_A(z)$ and their errors (computed with the Seo and Eisenstein procedure, see [41]), we fit the mock data to a 3rd (4th) grade polynomial for $H(z)$ ($d_A(z)$). This choice is motivated as follows. Even with extremely high-quality future data, only a reduced number of dark energy parameters can ever be measured e.g.,[43]. A general and flexible fitting function is thus a polynomial. Eigenmodes or bin-based approaches [44, 45] would effectively impose a drastic smoothing, erasing the signal even more. Here, the role of this parameterization is primarily to quantify the size of the error-bars of w_H and w_{d_A} achievable from future surveys. As it is clear from Fig. 4 the error-bars are much larger than the "signal" making the conclusions of this section rather insensitive to the type of parameterization used for $w(z)$.

With these polynomials we reconstruct $w_H(z)$ and $w_{d_A}(z)$ together with their errors and we present the results in Fig. 4. For $w_m = \Omega_m h^2$ we assume a 2% error, as expected from Planck data [46]. If the curvature is negative, the reconstructed $w(z)$ from $H(z)$ should have a divergence, while the $w(z)$ reconstructed from $d_A(z)$ should not diverge. The different behaviour for these two $w(z)$'s was already pointed out by the authors of Ref. [23].

However, in practice, this reconstruction procedure is not good enough to reproduce the divergence that would signal the presence of curvature. Indeed, as can be seen from Fig. 4, the errors with which the coefficients of the fitting polynomials would be reconstructed, even with the most optimistic survey assumed here, yield an allowed $w(z)$ region too large to see the expected signal. Moreover, the order of the fitting polynomials assumed is too low to track precisely enough the behaviour of $H(z)$ and $d_A(z)$ to the high redshifts at which the divergence occurs. Indeed, we find that for $z > 3$ the fit is not reliable, explaining why the best fit curves in Fig. 4 may miss (or find) a divergence when their theoretical counterparts of Fig. 3 (do not) display it.

We therefore conclude that, at least with the polynomial reconstruction method followed here, curvature and dynamical dark energy can not be disentangled by using the functions $w_H(z)$ and $w_{d_a}(z)$. It could be interesting to see if more refined parametric reconstruction approaches, or even non-parametric methods (see eg. [47]), could alleviate the $\Omega_k - w(z)$ degeneracy. This, at least at first sight, seems unlikely for forthcoming experiments: the error-bars on the reconstruction seems to be much larger than the signal in the redshift range accessible to future galaxy surveys. We can however still hope to be able to use a simple parameterization of $w(z)$ and to separate curvature from dark energy via a likelihood analysis from future BAO data.

4. Future constraints

We discuss here the forecasted errors on the dark energy equation of state expected from future surveys, in particular a survey with the characteristics of Planck [9] for the CMB and surveys with characteristics not too dissimilar from those of BOSS, Euclid/JDEM and LSST for BAO surveys, allowing for non-zero spatial curvature (for a related work see [48]).

The parameterization chosen for $w(z)$ is given by Eq. (2.3). We will assume that the fiducial model is ΛCDM, that is, $w_0 = -1$ and $w_a = 0$, but we will leave w_0 and w_a as free parameters in the fit.

The forecasted errors are estimated using the Fisher matrix formalism. For the CMB data, we have computed the Fisher matrix for a full-sky CMB experiment with the noise

[9] http://www.esa.int/Planck

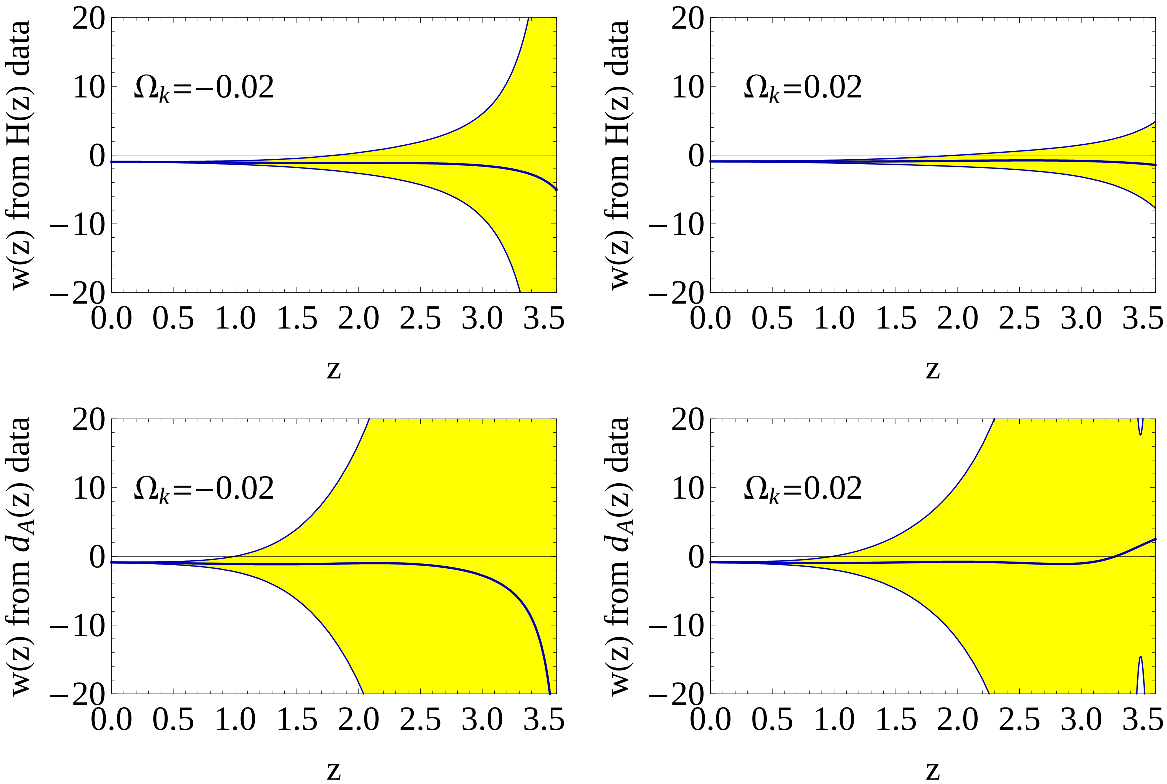

Figure 4: *The top (bottom) panels depict the reconstructed $w(z)$ from $H(z)$ ($d_A(z)$) best fit polynomial, see Eqs. (3.4) and (3.5). The left (right) figures illustrate the case of a negative (positive) curvature* $\mid \Omega_k \mid= 0.02$.

and resolution characteristics of the Planck survey. We used the following parameters for the Fisher matrix describing Planck data:

$$\theta = \{w_b, w_{dm}, \theta_{CMB}, \Omega_k, \tau, n_s, \alpha, A_s\} \ , \tag{4.1}$$

where $w_b = \Omega_b h^2$ and $w_{dm} = \Omega_{dm} h^2$ are the physical baryon and dark matter densities respectively, θ_{CMB} is proportional to the ratio of the sound horizon to the angular diameter distance, τ is the reionisation optical depth, n_s is the scalar spectral index, α is the running of the scalar spectral index and A_s the scalar amplitude.

We combine the CMB Fisher matrix with the BAO one for the large scale structure surveys. We build the Fisher matrix assuming measurements of $H(z)s$ and $d_A(z)/s$ in redshift bins of 0.1 width where s is the BAO scale, see Ref. [41] for a detailed description of the method used to estimate the errors on $H(z)s$ and $d_A(z)/s$. The characteristics and redshift intervals of the different surveys are shown in Tab. 1. BOSS and Euclid/JDEM are spectroscopic surveys and therefore the corresponding photo-z errors are set to zero. We illustrate two possible photo-z errors for the LSST-type survey (2% and 5%), encompassing optimistic and more realistic expectations.

In order to produce forecasts for the dark energy parameters and the spatial curvature

Survey	$n(h/Mpc)^3$	Area (square degrees)	Redshift range	σ_z
BOSS	3×10^{-4}	10000	$0.1 - 0.7$	0
Euclid/JDEM	1.9×10^{-3}	20000	$0.7 - 2.0$	0
LSST	3×10^{-3}	30000	$0.3 - 3.6$	2%, 5%

Table 1: Mean galaxy density, covered area, redshift range and photo-z error of the different surveys considered here.

we combine the two Fisher matrices after performing a transformation to the following set of parameters:$(H_0, \Omega_k, w_b, w_{dm}, w_0, w_a, \tau, n_s, \alpha, A_s)$. Notice that only the CMB Fisher matrix contains information regarding the last four parameters τ, n_s, α, and A_s. Finally we extract the contours in the w_0-Ω_k, w_a-Ω_k planes depicted in Figs. 5, 6, 7 and 8, marginalising over the other parameters.

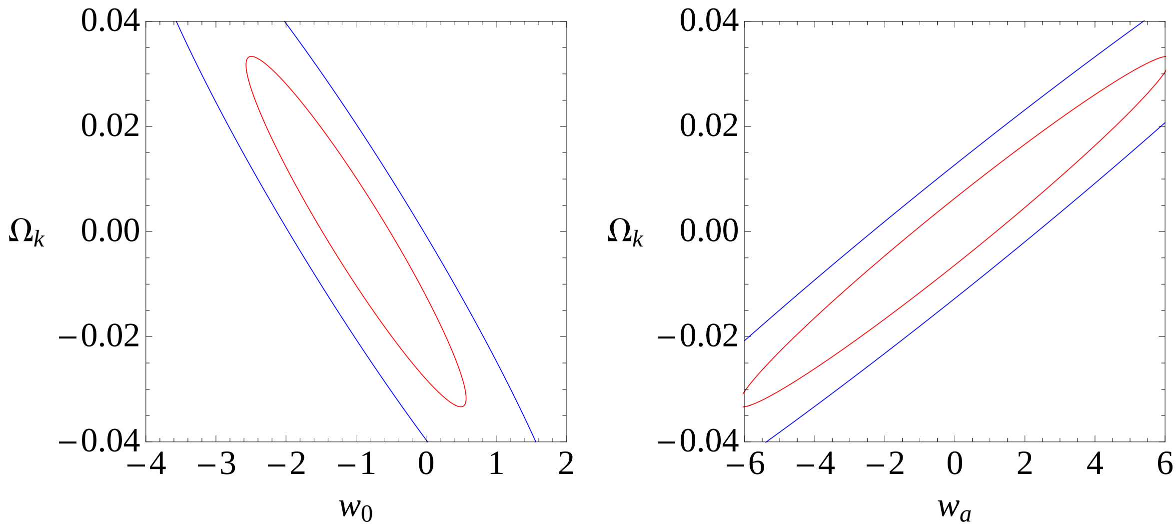

Figure 5: *Left (right) panel: 1 and 2 − σ forecasted errors from the BOSS-type + Planck-type surveys in the w_0-Ω_k (w_a-Ω_k) plane.*

Surveys limited to low-redshifts will not have enough statistical power to lift the dark-energy-curvature degeneracy, see Figs. 5. The situation may improve if future SNIa luminosity distance data were to be included. However SNe dataeffectively add statistical power only to the d_A constraint not to the $H(z)$ one. In fact d_L and d_a are both integrals of $H(z)$ and differ only by a normalization factor of $(1+z)^2$. As shown by Ref. [49] $H(z)$ is the key observable in disentangling $w(z)$ from Ω_k.

Euclid/JDEM-type data, with extended volume and redshift coverage, will improve significantly the current curvature constraints, see Figs. 6. A similar improvement could be provided by the photometric LSST survey if its photo-z $\sim 2\%$, see Figs. 7. The curvature could be constrained to the ~ 0.001 level and the constraints on the dark energy parameters could be improved by almost a factor four. Moreover, with its extended redshift coverage,

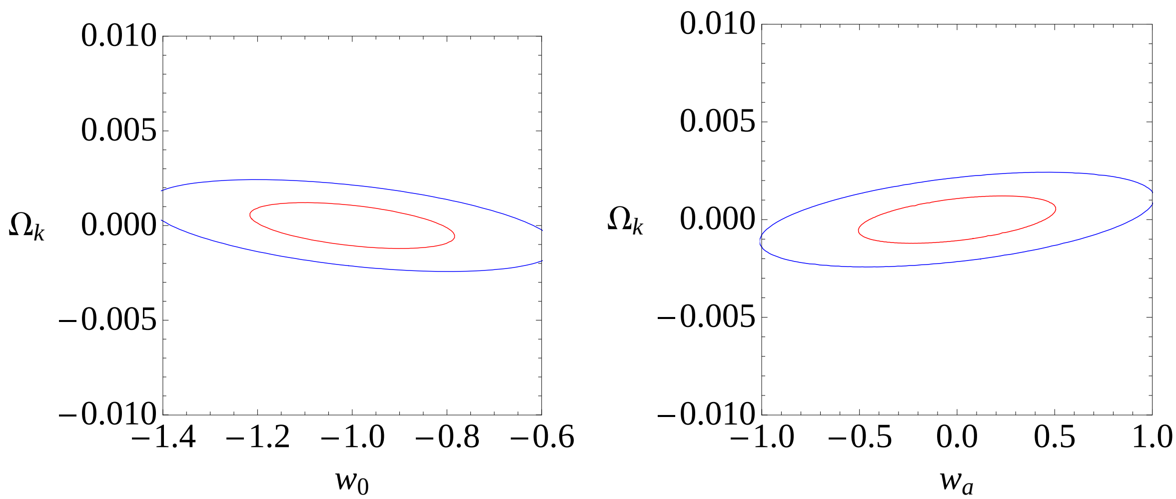

Figure 6: *Left (right) panel: 1 and $2-\sigma$ forecasted errors from the Euclid/JDEM-type + Planck-type surveys on the w_0-Ω_k (w_a-Ω_k) plane.*

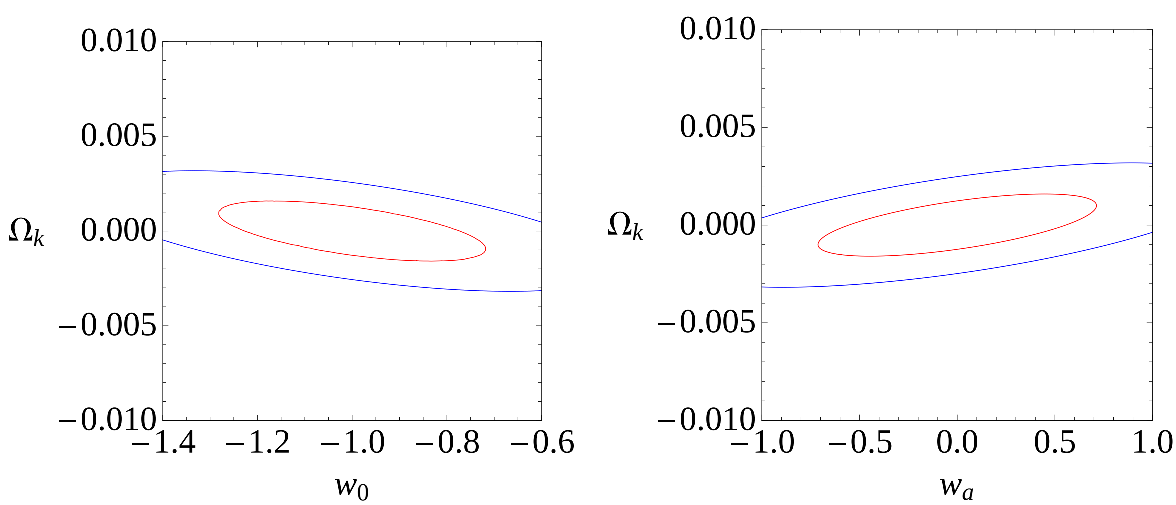

Figure 7: *Left (right) panel: 1 and $2-\sigma$ forecasted errors from the LSST + Planck surveys on the w_0-Ω_k (w_a-Ω_k) plane. A 2% photo-z error is assumed for the LSST survey.*

such a survey will yield almost no residual correlation between the dark energy equation of state parameters and curvature, as can be seen in Figs. 6 and 7. If the photo-z error is higher, $\sigma_z \sim 5\%$, the error in the curvature is still at the ~ 0.001 level but the error on the dark energy parameters w_0 and w_a increases considerably, see Figs. 8. The reason for that is because a higher photo-z error will suppress exponentially the radial BAO modes and therefore the information on $H(z)$ (which is crucial for the measurement of $w(z)$) will be completely lost.

In Figs. 9 we investigate the effect of a survey's redshift coverage on the parameter's errors. We show the 1σ marginalised error on w_0, w_a and Ω_k expected from Euclid/JDEM-

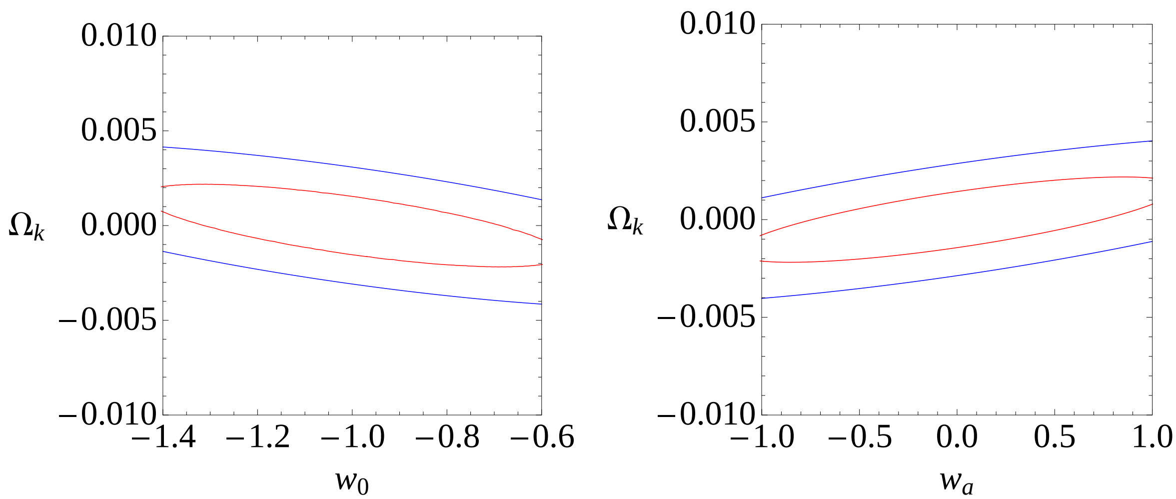

Figure 8: *Left (right) panel: 1 and $2-\sigma$ forecasted errors from the LSST + Planck surveys on the w_0-Ω_k (w_a-Ω_k) plane. A 5% photo-z error is assumed for the LSST survey.*

type and LSST-type surveys versus the maximum redshift (assuming, quite unrealistically, that such surveys could actually reach such high redshifts).

Notice from those figures that there exists a *critical* redshift $z_{cr} \sim 2-3$ beyond which the marginalised errors on w_0, w_a and Ω_k do not improve significantly. The z_{cr} can be understood if we consider that there is a redshift $\hat{z}$ at which the derivative of the angular diameter distance with respect to the curvature changes sign [19]:

$$\left.\frac{\partial d_A(z=\hat{z})}{\partial \Omega_k}\right|_{\Omega_k=0} = 0 \ , \tag{4.2}$$

being $d_A(z)$ negative (positive) for redshifts below (above) $\hat{z}$.

For a ΛCDM cosmology with $h = 0.72$ and $\omega_m = 0.12$, $\hat{z} = 3.2$. Planck data will provide an exquisite measurement of d_A at a redshift $z \sim 1088$, i.e. well above $\hat{z}$. One would thus need to measure with a good precision $d_A(z)$ below $\hat{z}$ to break the dark energy-curvature degeneracy. For doing that, the maximal redshift covered by the survey should not be very far from $z = \hat{z}$. For the chosen parameterization of the dark energy equation of state, going to higher redshifts, $z \gg 2$, is unnecessary, since the errors on w_0, w_a and Ω_k will not be significantly reduced further. Of course, this conclusion depends on the assumed $w(z)$ parameterization.

5. Conclusions

In this work we have assessed critically the prospects for disentangling the degeneracy between a non-constant dark energy equation of state parameter $w(z)$ and a non zero curvature Ω_k. We have considered constraints achievable from surveys with characteristics not too dissimilar from those of proposed baryon Acoustic Oscillation galaxy surveys. We have shown that despite the spectacular resonant-like behaviour exhibited by the $w(z)$

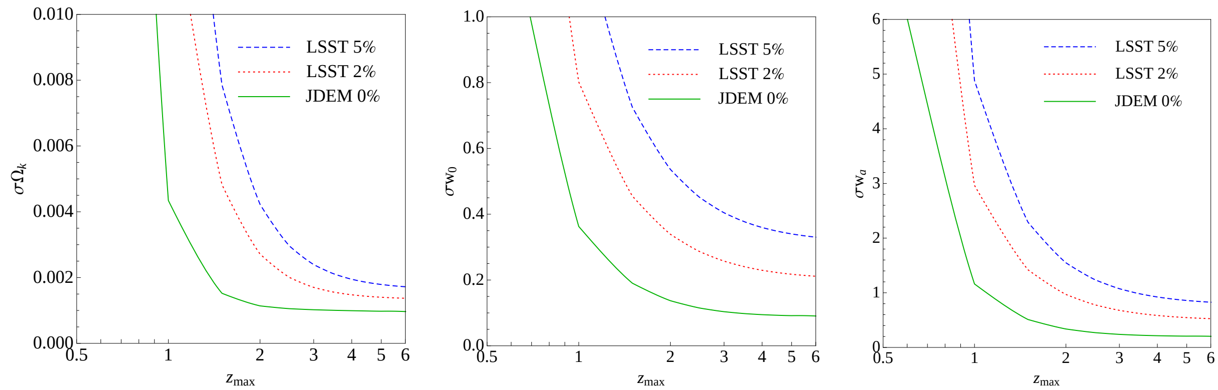

Figure 9: *1σ marginalised error on w_0, w_a and Ω_k expected from Euclid/JDEM-type and LSST-type surveys versus the redshift coverage (maximum redshift). For the LSST-type survey, we illustrate the results assuming two different photo-z errors, 2% and 5%.*

needed to match $H(z)$- $d_A(z)$ when erroneously assuming a flat universe, the degeneracy cannot be solved in a model-independent way from realistic observations. Nevertheless we have shown that it is possible from future data to disentangle curvature from dark energy evolution if a dark energy parameterization is assumed and by combining BAO data with CMB constraints. Adopting the popular two-parameter w_0, w_a parameterization of the redshift evolution of the dark energy equation of state parameter we find that measurements of both $H(z)$ and $d_A(z)$ up to sufficiently high redshifts $z \sim 2$ is key to resolve the $w(z) - \Omega_k$ degeneracy. These results are in qualitative agreement with e.g. Ref. [20]. The agreement is not quantitative but this is due to the fact that we consider BAO (transversal and radial) while Ref. [20] concentrates on SNe. Here we have focused on BAO surveys rather than SNe data for several reasons. The luminosity distance d_L (obtained from SNe data) is an integral over $H(z)$ as it is d_A: adding SNe data wouldreduce the error-bars around w_{d_A} but not around w_H. For parametrizations of the dark energy equation of state more general than the one in Eq. (2.2), it has been shown that non-integrated quantities such as $H(z)$ allow for a better reconstruction of the dark energy dynamics than distance measurements such as d_A or d_L, see eg. Refs. [47, 49]. Finally, even when the simple parametrization of Eq. (2.2) is assumed, we have found that d_A and $H(z)$ measurements atsufficiently high redshifts, around $z \sim 2$, are required to solvethe $w(z) - \Omega_k$ degeneracy; currently planned SNe surveys do not reach such high redshifts. While quantitatively the results presented here will improve with the addition of SNe data, we believe we have captured qualitatively the gist of forthcoming constraints.

The conclusions presented here depend quantitatively on the parameterization chosen, and, probably to a lesser extent, on the fiducial model adopted. However, qualitatively the conclusions should hold in general; it is always possible to find a better parameterization that will yield smaller errors, but at least we have shown that with the parameterization adopted here forthcoming surveys will lift the curvature/dark energy degeneracy (see fig 7 and 8). This conclusion, however,relies on the fact that the adopted parameterization includes the underlying model. Should there be an indicationof a deviation from constant

$w(z)$, the issue of the $w(z)$ parameterization will become crucial for obtaining meaningful results.

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